VIVEK TUTORIALS

X (English)

(Chapter One)

Mathematics Part - II-(Chapter One)

DATE: 20-02-19

TIME: 1 Hr

MARKS: 30

SEAT NO:

Q.1 Multiple Choice Questions 2

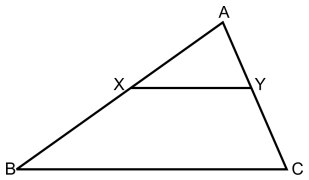
1 $\triangle ABC \sim \triangle PQR$. If A ($\triangle ABC$) = 25, ($\triangle PQR$) = 16, find AB : PQ.

a. 25 : 16 b. 4 : 5 c. 16 : 25 d. 5 : 4

Ans 5 : 4

2 In figure, seg XY || seg BC, then which of the following statements is true?

a. $\frac{AB}{AC}=\frac{AX}{AY}$ b. $\frac{AX}{XB}=\frac{AY}{AC}$ c. $\frac{AX}{YC}=\frac{AY}{XB}$ d. $\frac{AB}{YC}=\frac{AC}{XB}$



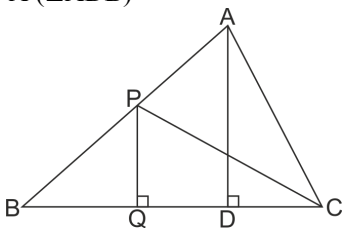
Ans Option a

Q.2 Solve the following 4

1 In the adjoining figure, PQ ⊥ BC, AD ⊥ BC, PQ = 4, AD = 6.

Write down the following ratios.

$\frac{A\ (\triangle PQB)}{A\ (\triangle ADB)}$



Ans In ΔPQB and ΔADB,

$\angle B \cong \angle B$

$\angle PQB \cong \angle ADB$ ... (each right angle)

$\therefore$ $\Delta PQB \sim \Delta ADB$ ... (A-A test of similarity)

$\therefore$ $\frac{A\ (\triangle PQB)}{A\ (\triangle ADB)} = \frac{PQ^2}{AD^2} = \frac{4^2}{6^2} = \frac{16}{36} = \frac{4}{9}$ ... (Theorem of areas of similar triangle)

2 In trapezium ABCD, side AB || side CD, diagonal AC and BD intersect each other at point P. Then prove that

$\frac{A(\triangle ABP)}{A(\triangle CPD)}=\frac{AB^2}{CD^2}$

Ans In trapezium ABCD side AB || side CD

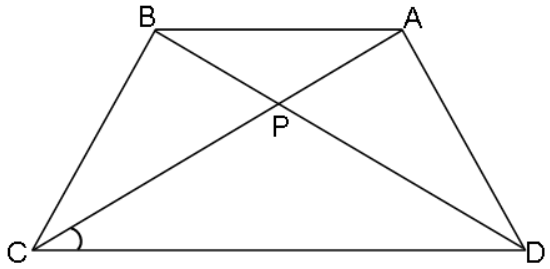
In $\triangle APB$ and $\triangle CPD$

$\angle PAB \cong \angle PCD$ ... alternate angles

$\angle APB \cong \angle CPD$ ... opposite angles

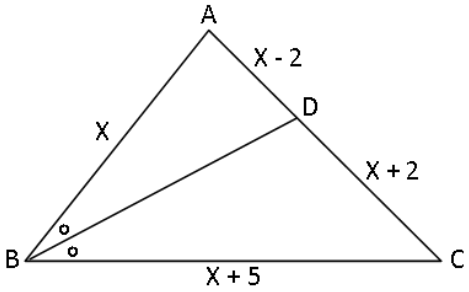
$\therefore$ $\triangle APB \sim \triangle CPD$ ... AA test of similarity

$\frac{A(\triangle ABP)}{A(\triangle CPD)} = \frac{AB^2}{CD^2}$ ... theorem of areas of similar triangles



Q.3 Answer the following 4

1 In △ ABC, seg BD bisects ∠ ABC. If AB = x, BC = x + 5, AD = x – 2, DC = x + 2, then find the value of x.



Ans In △ ABC

ray BD is the bisector of ∠ ABC

$\therefore \quad \frac{BA}{BC} = \frac{AD}{CD}$ ... (Angle bisector property)

$\frac{x}{x+5} = \frac{x-2}{x+2}$

$x(x+2) = (x+5)(x-2)$

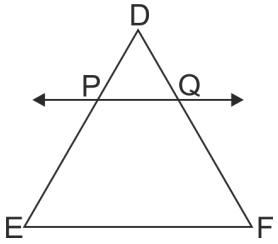
$x^2 + 2x = x^2 + 5x - 2x - 10$

$\therefore \quad 2x = 3x - 10$

$\therefore \quad x = 10$

$\therefore$ The value of x is 10

2 In △DEF, line PQ || side EF, if DP = 2.4, PE = 7.2, DQ = 1.8 then find QF.



Ans In △DEF, line PQ || side EF

$\therefore \quad \frac{DP}{PE} = \frac{DQ}{QF}$

$\therefore \quad \frac{2.4}{7.2} = \frac{1.8}{QF}$

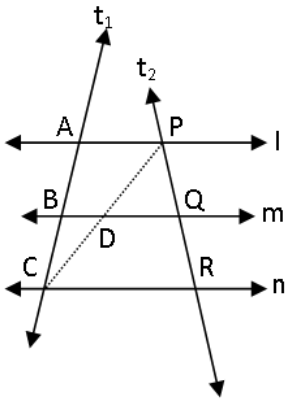
$\therefore \quad QF = \frac{7.2 \times 1.8}{2.4}$

$\therefore \quad QF = 5.4$

Q.4 Solve the following 6

1 Prove that: The ratio of the intercepts made on a transversal by three parallel lines is equal to the ratio of the corresponding intercepts made on any other transversal by the same parallel lines.

Ans



Given : line l || line m || line n t_1 and t_2 are transversals. Transversal t_1 intersects the lines in points A, B, C and t_2 intersects the lines in points P, Q, R.

To prove : $\frac{AB}{BC} = \frac{PQ}{QR}$

Proof : Draw seg PC , which intersects line m at point D.

In $\triangle ACP$, BD || AP

$\therefore \frac{AB}{BC} = \frac{PD}{DC}$ ... (I) (Basic proportionality theorem)

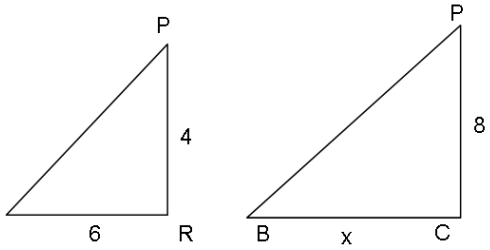
In $\triangle CPR$, DQ || CR

$\therefore \frac{PD}{DC} = \frac{PQ}{QR}$ ... (II) (Basic proportionality theorem)

$\therefore \frac{AB}{BD} = \frac{PD}{DC} = \frac{PQ}{QR}$ ... from (I) and (II).

$\therefore \frac{AB}{BC} = \frac{PQ}{QR}$

2 As shown in figure, two poles of height 8 m and 4 m are perpendicular to the ground. If the length of shadow of smaller pole due to sunlight is 6 m then how long will be the shadow of the bigger pole at the same time ?



Ans $\therefore$ The shadow of both the poles are formed at the same time.

$\therefore \frac{\text{The length of the smaller pole}}{\text{The length of the bigger pole}} = \frac{\text{Height of the shadow of smaller pole}}{\text{Height of the shadow of bigger pole}}$

$\therefore \frac{4}{8} = \frac{6}{x}$

$\therefore 4 \times x = 8 \times 6$

$\therefore x = \frac{8 \times 6}{4}$

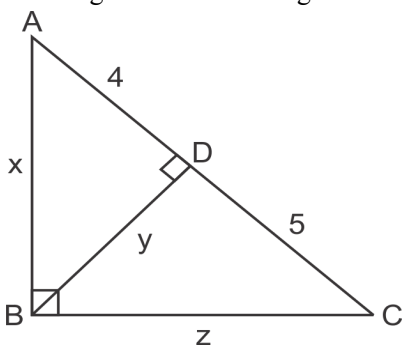
$\therefore x = \frac{48}{4}$

$\therefore x = 12$ m

$\therefore$ Corresponding height of the shadow of the bigger pole is 12m.

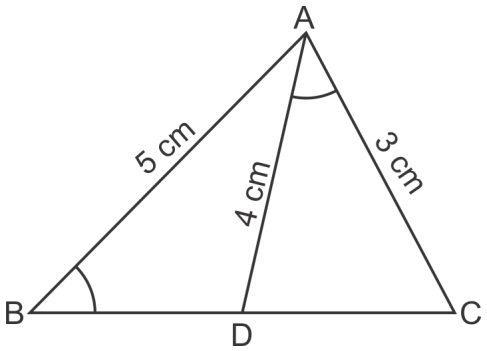
Q.5 Answer the following 8

1 In the given figure, an altitude is drawn to the hypotenuse.
The lengths of different segment are marked in the figure, determine the value of x, y, z



Ans (1)In $\angle ABC$, $\angle B = 90°$ ... (given)
seg BD $\perp$ hypoteuse AC
$\therefore$ $BD^2 = AD \times DC$ ... (Geometric mean property)
(2)But,
BD = y, AD = 4, DC = 5 ... (given)
(3)$\therefore$ $y^2 = 4 \times 5$ ... [From (1) and (2)]
$\therefore$ $y = \sqrt{4 \times 5}$ ... (Taking sq. root)
(4)In $\triangle ADB$, $\angle ADB = 90°$, ... (given)
$\therefore$ $AB^2 = AD^2 + BD^2$ ... (Pythagoras theorem)
(5)But, AB = x ... (given)
$\therefore$ $x^2 = (4)^2 + y^2$ ... [From (2), (4) and (5)]
$\therefore$ $x^2 = 16 + (2\sqrt{5})^2$ ... [From (3)]
$\therefore$ $x^2 = 16 + 4 \times 5$
$\therefore$ $x^2 = 16 + 20$
$\therefore$ $x^2 = 36$
$\therefore$ x = 6 units ... (Taking sq. root)
In $\triangle BDC$, $\angle BDC = 90°$ ... (Given)
$\therefore$ $BC^2 = BD^2 + DC^2$ ... (Pythagoras theorem)
But, BC = z, BD = y, DC = 5... (Given)
$\therefore$ $z^2 = y^2 + 5^2$ ... [From (7) and (8)]
$\therefore$ $z^2 = (2\sqrt{5})^2 + 5^2$ ... [From (3)]
$\therefore$ $z^2 = (4 \times 5) + 25$
$\therefore$ $z^2 = 20 + 25$
$\therefore$ $z^2 = 45$
$\therefore$ $z = \sqrt{45}$
$\therefore$ $z = \sqrt{9 \times 5}$ ... (Taking sq. root)
$\therefore$ $z = 3\sqrt{5}$
x = 6 units $y = 2\sqrt{5}$ units, $z = 3\sqrt{5}$ units

2 In the adjoining figure, D is a point on side BC such that $\angle ABD = \angle CAD$.
If AB = 5m, AD = 4 cm, and AC = 3 cm.
Find : (i) BC, (ii) DC (iii) $A(\triangle ACD) : A(\triangle BCA)$



Ans Proof:

(1)$\angle ABD \cong \angle CAD$ ... (Given)

i.e. $\angle ABC \cong \angle CAD$ ... (B-D-C)

(2)In $\triangle ABC$ and $\triangle DAC$

For $ABC \leftrightarrow DAC$

(a) $\angle ABD \cong \angle CAD$ ... [from (1)]

(b) $\angle C \cong \angle C$ ... (common angle)

(c) $\therefore \quad \triangle ABC \sim \triangle DAC$ ... (A - A Test of similarity)

(3)$\therefore \quad \frac{AB}{DA} = \frac{AC}{DC} = \frac{BC}{AC}$ ... (c.s.s.t)

(4)But, AB = 5 cm, AD = 4 cm, AC = 3 cm ...(given)

(5)$\therefore \quad \frac{5}{4} = \frac{3}{DC} = \frac{BC}{3}$ ...[from (3) and (4)]

$\therefore \quad DC = \frac{12}{5}$

$= \frac{12}{5}$

$\therefore \quad DC = 2.4$ cm

Also, $\frac{5}{4} = \frac{BC}{3}$

$\therefore \quad BC = \frac{5 \times 3}{4}$

$BC = \frac{15}{4}$

$\therefore \quad BC = 3.75$ cm

(6)$\frac{A(\triangle ACD)}{A(\triangle BCA)} = \frac{AD^2}{AB^2}$ ... [Theorem on areas of similar triangles]

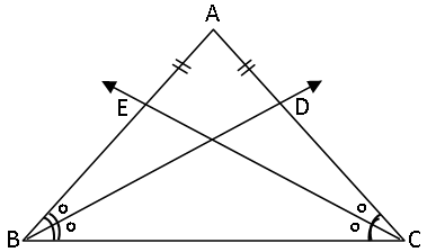
$\therefore \quad \frac{A(\triangle ACD)}{A(\triangle BCA)} = \frac{4^2}{5^2}$ ... [From (4)]

$\frac{A(\triangle ACD)}{A(\triangle BCA)} = \frac{16}{25}$

(1) BC = 3.75 cm, (2) DC = 2.4 cm, (3) $\therefore A(\triangle ACD) : A(\triangle BCA) = 16 : 25$

Q.6 Answer the following 6

1 In $\triangle ABC$, ray BD bisects $\angle ABC$ and ray CE bisects $\angle ACB$. If seg AB $\cong$ seg AC then prove that ED || BC.



Ans Construction : Draw seg ED.

1) In ΔABC, ray BD bisects ∠ABC.... [Given]

∴ $\frac{AD}{DC} = \frac{AD}{DC}$ ... [Angle bisector theorem of a triangle]

2) In ΔABC, ray CE bisects ∠ACB. ... [Given]

∴ $\frac{AE}{EB} = \frac{AC}{BC}$ ... [Angle bisector theorem of a triangle]

3) $\frac{AE}{EB} = \frac{AC}{BC}$ ... [from (2), seg AB≅seg AC, given]

4) $\frac{AD}{DC} = \frac{AE}{EB}$ ... [from (1), (3)]

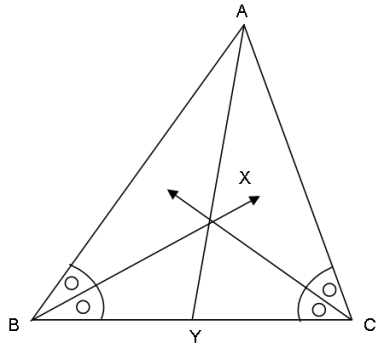
5) In ΔABC

$\frac{AE}{EB} = \frac{AD}{DC}$ ... [from (4)]

∴ seg ED ‖ BC ... [Converse of Basic proportionality theorem]

i.e. ED‖ BC.

2 In bisectors of ∠B and ∠C of Δ ABC intersect each other in point X. Line AX intersects side BC in point Y. AB = 5, AC = 4, BC = 6 then find $\frac{AX}{XY}$.



Ans In Δ AYV

Seg BX is a bisector of ∠ ABY

∴ $\frac{AB}{BY} = \frac{AX}{XY}$ (1) Angle bisector property)

In Δ AYC

Seg CX is the bisector of ∠ ACY

∴ $\frac{AC}{CY} = \frac{AX}{XY}$ (2) Angle bisector property)

∴ from (1) & (2)

$\frac{AB}{BY} = \frac{AC}{CY} = \frac{AX}{XY}$

By theorem on equal ratios.

$\frac{AB + AC}{BY + CY} = \frac{AX}{XY}$

$\frac{AB + AC}{BC} = \frac{AX}{XY}$

$\frac{5 + 4}{6} = \frac{AX}{XY}$

$\frac{9}{6} = \frac{AX}{XY}$

$\frac{3}{2} = \frac{AX}{XY}$

∴ $\boxed{\frac{AX}{XY} = \frac{3}{2}}$